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MIMO Capacity of Radar as a Communications Channel

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Adaptive Sensor and Array Processing Workshop
11 - 13 March 2003

Work funded under DARPA/AFRL Digital RF Tags contract
Program Manager: Dr. Tim Grayson

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Radar as a Communications Channel

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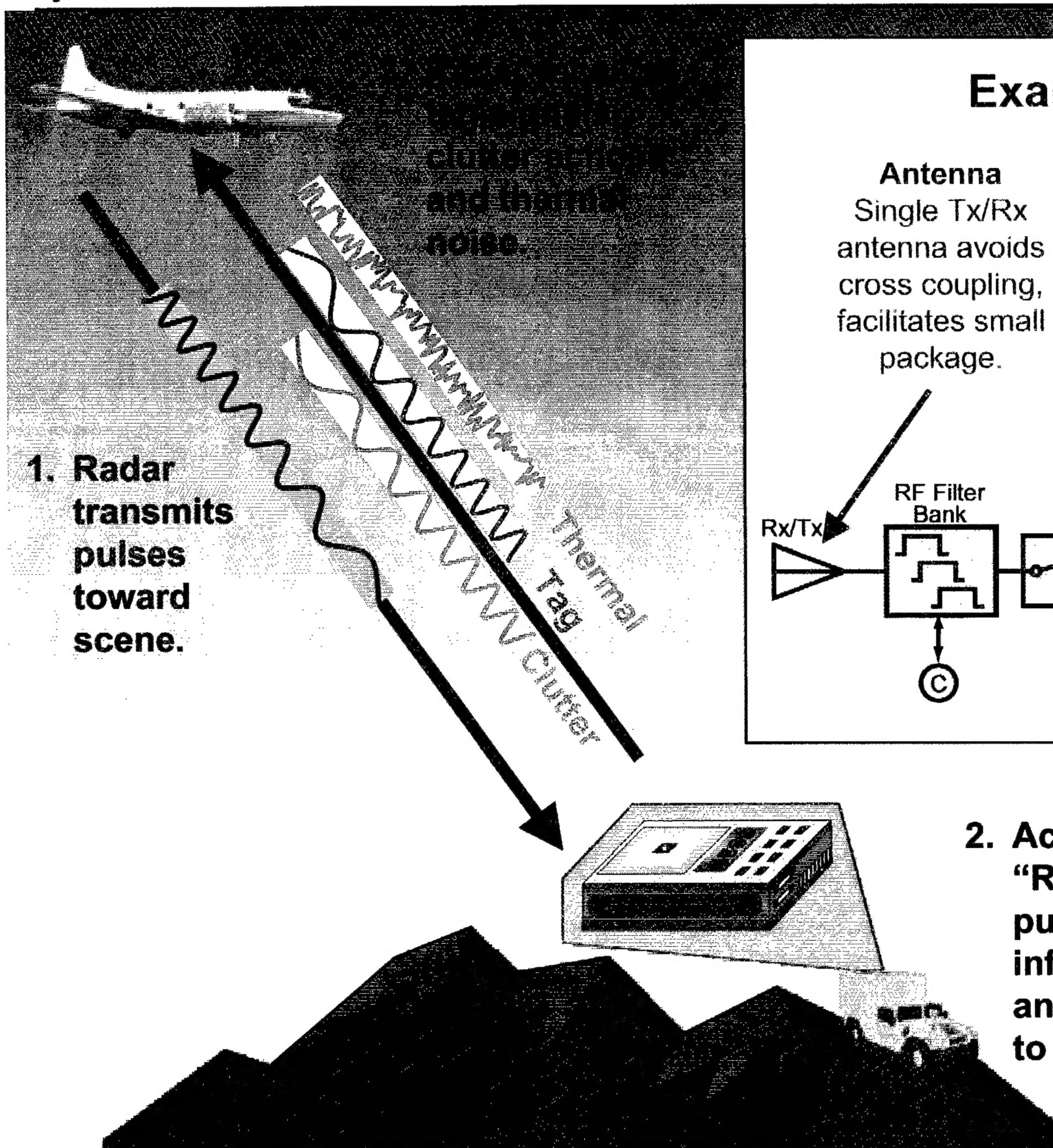


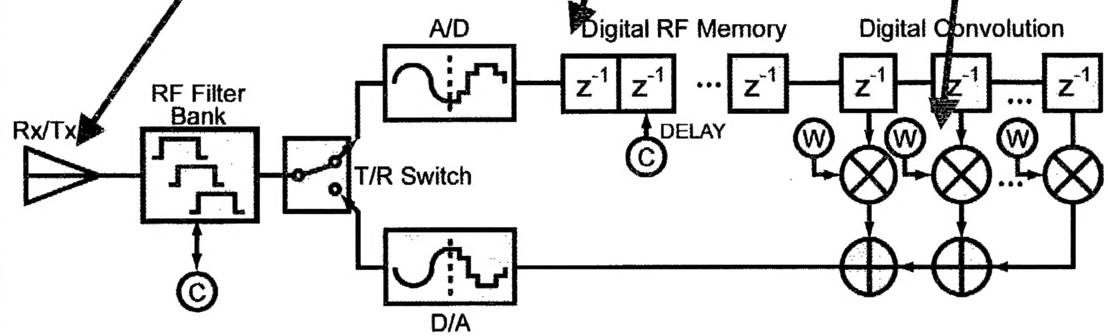
1. **Radar transmits pulses toward scene.**

Example RF Tag Architecture

Antenna
Single Tx/Rx antenna avoids cross coupling, facilitates small package.

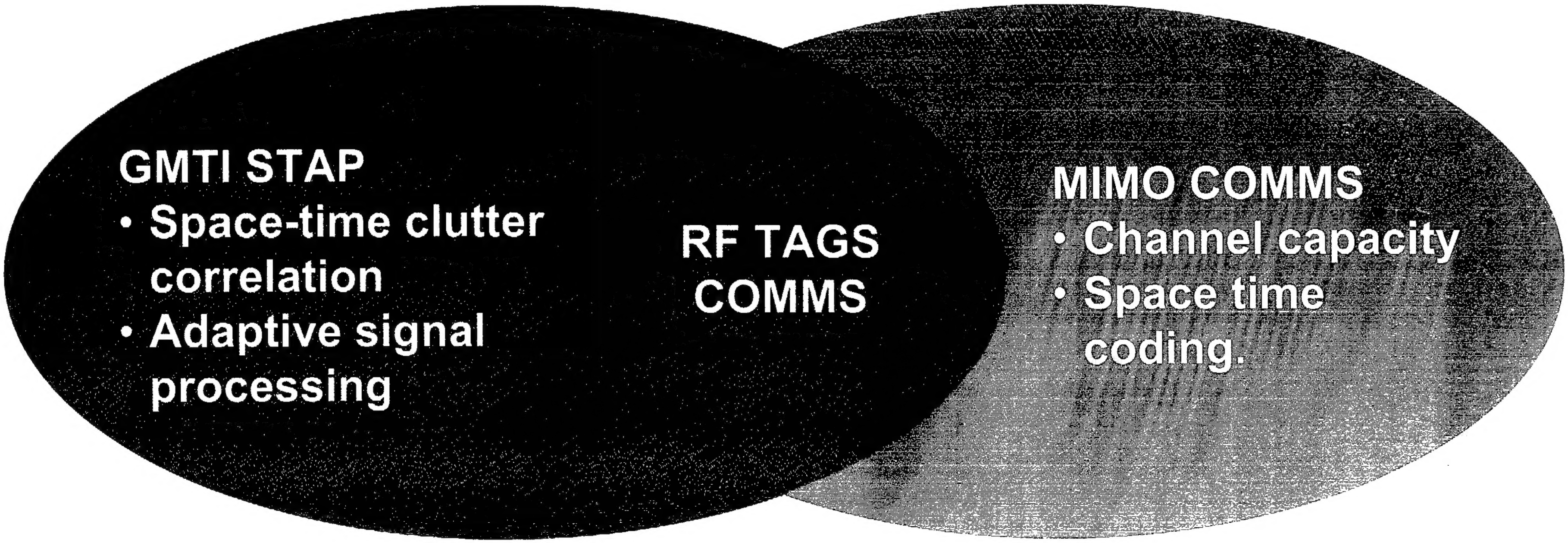
Digital RF Memory
Allows range ambiguous operation: n^{th} pulse is transponded on $n+1^{st}$ PRI.

Fast-Time Convolver
Coded data is convolved onto incident radar pulses.



2. **Active transponder "RF tag" captures pulses, encodes information onto these and retransmits back to radar.**

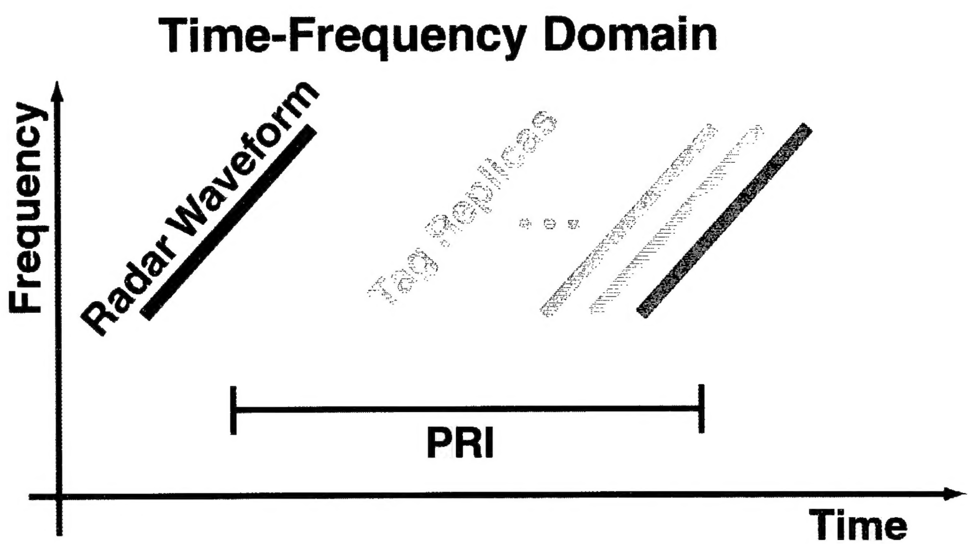
RF Tags Communications

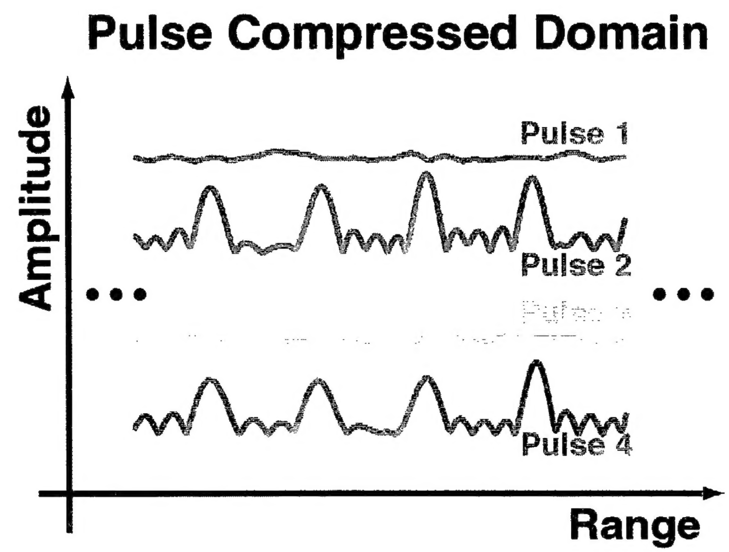


Results Summary

- **ASAP 2002: Adaptive signal processing algorithm for suppressing clutter while preserving tag signals in multichannel radar systems.**
- **Asilomar 2002: Shannon capacity bounds + example curves for single channel radar systems.**
- **ASAP 2003: Shannon capacity bounds + example curves for multichannel radar systems.**

RF Tag Signal Models





- Tag convolver produces a weighted sequence of time-delayed replicas of the radar waveform.
- Pulse compression causes each replica to compress in range.
- The impulse response peak values are determined by convolver tap weights.
- We consider *two* tag signal models:
  - Tag retransmits *all* received pulses.
  - Tag multiplexes single antenna to retransmit *every other* pulse.

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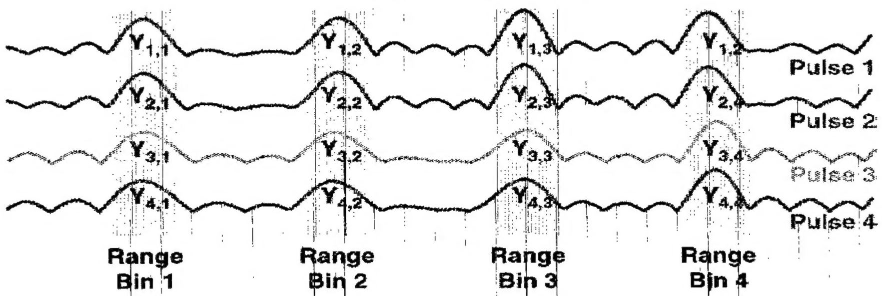
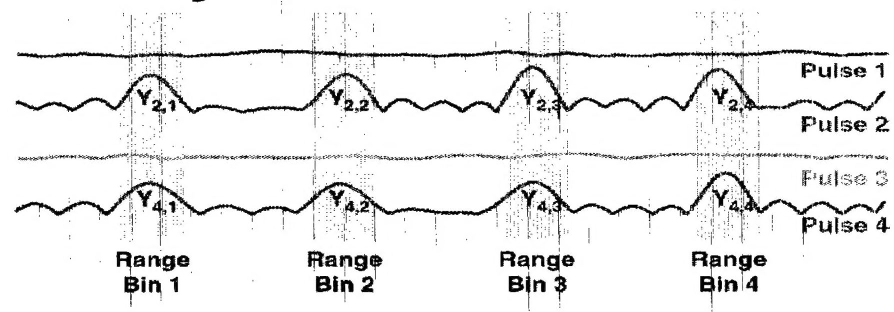
Radar Communications Channel

Single Receiver

All Pulse Model



Every Other Pulse Model



Channel Model

$$\mathbf{Y}_{n,k} = \mathbf{X}_{n,k} + \mathbf{Z}_{n,k}$$

$X_{n,k}$ - Input symbols n - Pulse number
$Z_{n,k}$ - Noise samples k - Range bin index
$Y_{n,k}$ - Output symbols N_{bins} - # Range bin channels

Noise Model

$$R_Z(n) = E\left[\mathbf{Z}_{m,k}\mathbf{Z}^*_{m+n,k}\right]$$ - WSS Autocorrelation

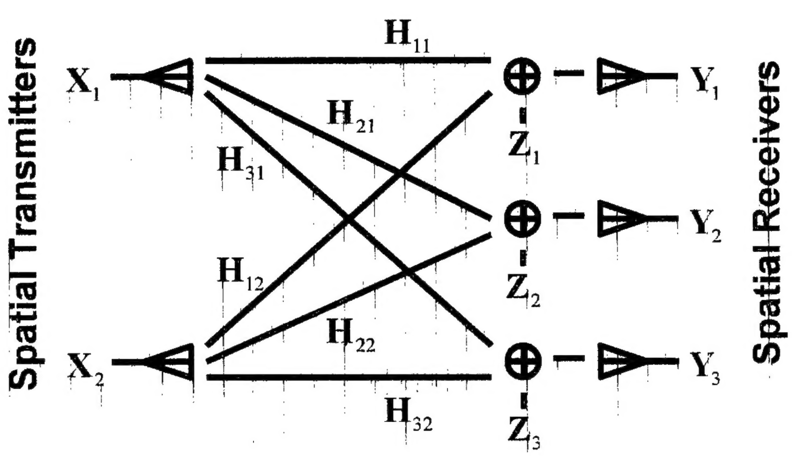
Clutter will be correlated between pulses.

- **We model the RF tag channel as a set of N_{bins} identical parallel discrete complex Gaussian channels corresponding to the tag signal bearing range bins.**
- **Each channel is contaminated with independent (range bin to range bin) additive WSS thermal and clutter noise.**

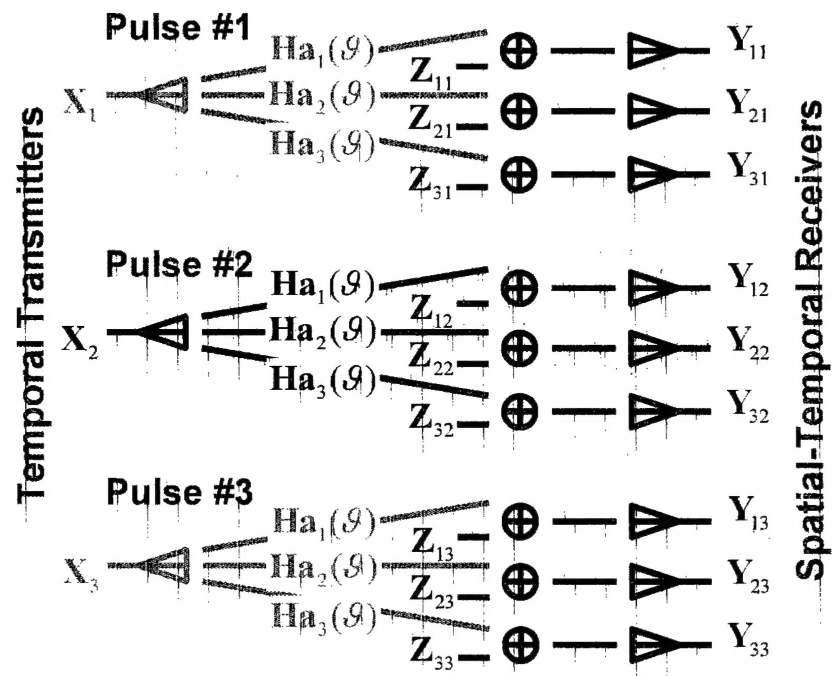
Radar Communications Channel

Multiple Receivers

Conventional MIMO



RF Tags



	Conventional MIMO	RF Tags
Channel Matrices	• Arbitrarily sized. • Arbitrary elements.	• More RX than TX. • Kronecker product of identity matrix with steering vector.
Interference	• Usually spatially white.	• Highly colored between receivers and pulses.
Time Samples	• Continuous	• Finite # of range bins within each pulse.

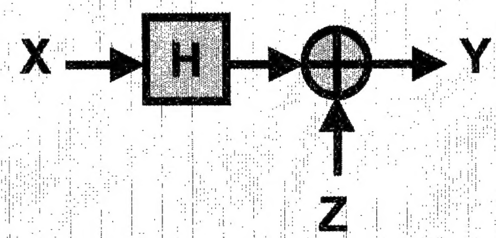
Injective* MIMO Channel Capacity

MIMO Channel Equation

$$\mathbf{Y} = \mathbf{HX} + \mathbf{Z}$$

$$\mathbf{H} - N_{RX} \times N_{TX}$$
$$\text{rank}(\mathbf{H}) = N_{TX}$$

Channel Diagram



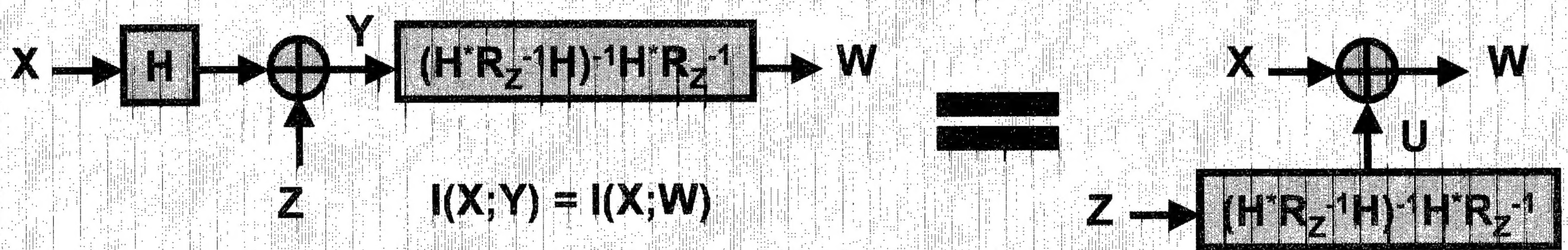
The MVUE of **X** given **Y** is:

$\mathbf{W} = (\mathbf{H}^*\mathbf{R}_Z^{-1}\mathbf{H})^{-1}\mathbf{H}^*\mathbf{R}_Z^{-1}\mathbf{Y}$

This is a sufficient statistic (receiver) for **X**, thus the cascaded channel has the same capacity:

I(X;Y) = I(X;W)

Cascaded Channel Diagram



I(X;Y) = I(X;W)

Conclusion: The injective MIMO channel has the same capacity as the bijective channel with identity channel matrix and noise process:

$$\mathbf{U} = (\mathbf{H}^*\mathbf{R}_Z^{-1}\mathbf{H})^{-1}\mathbf{H}^*\mathbf{R}_Z^{-1}\mathbf{Z}$$

***General MIMO Capacity**: Bliss et. al. Environmental issues for MIMO capacity. IEEE Trans. on Signal Processing, 50(9):2129-2142, Sept. 2002

Spectral Efficiency Formulas

Informed Transmitter Spectral Efficiency

Transmitter knows noise correlation statistics. Coded signal is spectrally optimized to maximize data rate.

$$C_{IT} = \sum_{i=1}^{N_{TX}} \log\left(\frac{(\nu - \lambda_i)^+ + \lambda_i}{\lambda_i} \right)$$

where ν satisfies the energy constraint:

$$\frac{1}{N_{TX}} \sum_{n=1}^{N_{TX}} (\nu - \lambda_i)^+ = E$$

Uninformed Transmitter Spectral Efficiency

Transmitter does not know correlation statistics. Optimal coded signal is spectrally white.

$$C_{UT} = \sum_{i=1}^{N_{TX}} \log\left(\frac{E + \lambda_i}{\lambda_i} \right)$$

Notation

N_{TX} - Number of transmit channels.

E - Energy per transmission.

$\{\lambda_1, \ldots, \lambda_{N_{TX}}\}$ - Eigenvalues of the equivalent bijective channel covariance matrix $\mathbf{R}_\mathrm{U} = (\mathbf{H}^* \mathbf{R}_\mathrm{Z}^{-1} \mathbf{H})^{-1}$

RF Tags Capacity Roadmap:

- Define spectral efficiency for the RF tags channel.
- Formulate the RF Tags channel transfer matrix H and energy constraint *E*.
- Determine the space-time interference covariance matrix R_Z as a function of the radar parameters.

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RF Tags MIMO Channel

Spectral Efficiency & Channel Capacity

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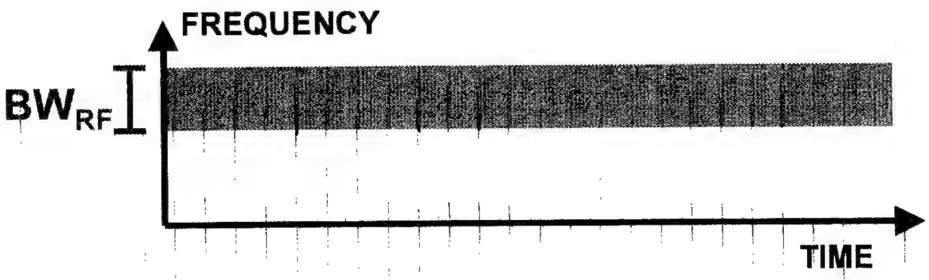
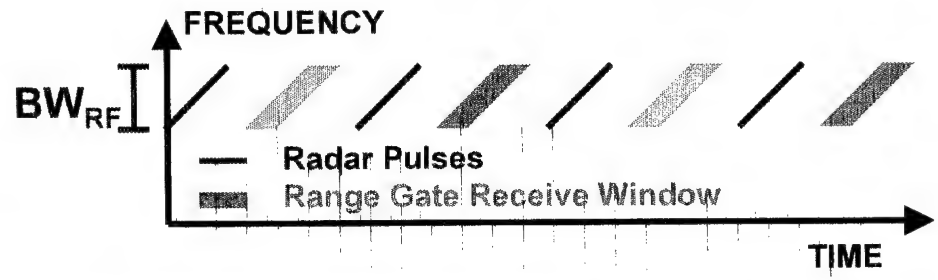
Conventional MIMO:

Continuous Time-Frequency Support



RF Tags:

Radar Time-Frequency Support



	Conventional MIMO	RF Tags
Time Frequency Support	RF band sampled directly at (or above) Nyquist rate. Full time frequency support available for communications.	Pulsed radar operation and front end processing allow only a small fraction of time frequency support to be utilized for communications.
Spectral Efficiency	Spectral efficiency is # bits per Nyquist sample.	Spectral efficiency is # bits per range bin per pulse.
Channel Capacity	Channel capacity is spectral efficiency times bandwidth.	Channel capacity is spectral efficiency times N_{bins} times PRF (or PRF/2).

RF Tags MIMO Channel

Energy Constraint & Transfer Matrix

Tag Energy

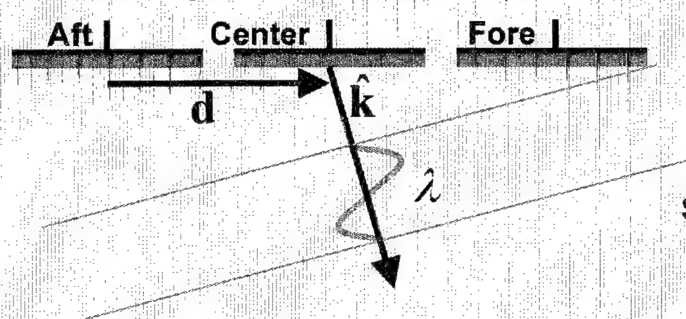
(Friis transmission equation)

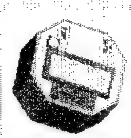
$$N_{bins} \times E_{tag} = ERP_{tag} \times T_p \times \frac{1}{4\pi R^2} \times \frac{\lambda^2 G_{tag}^{Rx}}{4\pi}$$

- N_{bins}: Range Bins Used
- E_{tag}: Per Bin Tag Energy
- ERP_{tag}: Tag Effective Radiated Power
- T_p: Pulse Width
- $\frac{1}{4\pi R^2}$: Propagation Loss to Radar
- $\frac{\lambda^2 G_{tag}^{Rx}}{4\pi}$: Radar Antenna Effective Area

Channel Transfer Matrices

Multiple Receiver Radar





M = # Pulses

N = # Receivers

Spatial Frequency

$$\vartheta = \frac{\hat{\mathbf{k}} \cdot \mathbf{d}}{\lambda}$$

Spatial Steering Vector

$$\mathbf{a}(\vartheta) = \begin{bmatrix} 1 \\ e^{2\pi i\vartheta} \\ e^{4\pi i\vartheta} \end{bmatrix}$$

Transpond All Pulses

$$\mathbf{H}_{AP} = \begin{bmatrix} \mathbf{a}(\vartheta) & \mathbf{0} & \Lambda \\ \mathbf{0} & \mathbf{a}(\vartheta) & \Lambda \\ \mathrm{M} & \mathrm{M} & \mathrm{O} \end{bmatrix} \quad MN \times M$$

Transpond Every Other Pulse

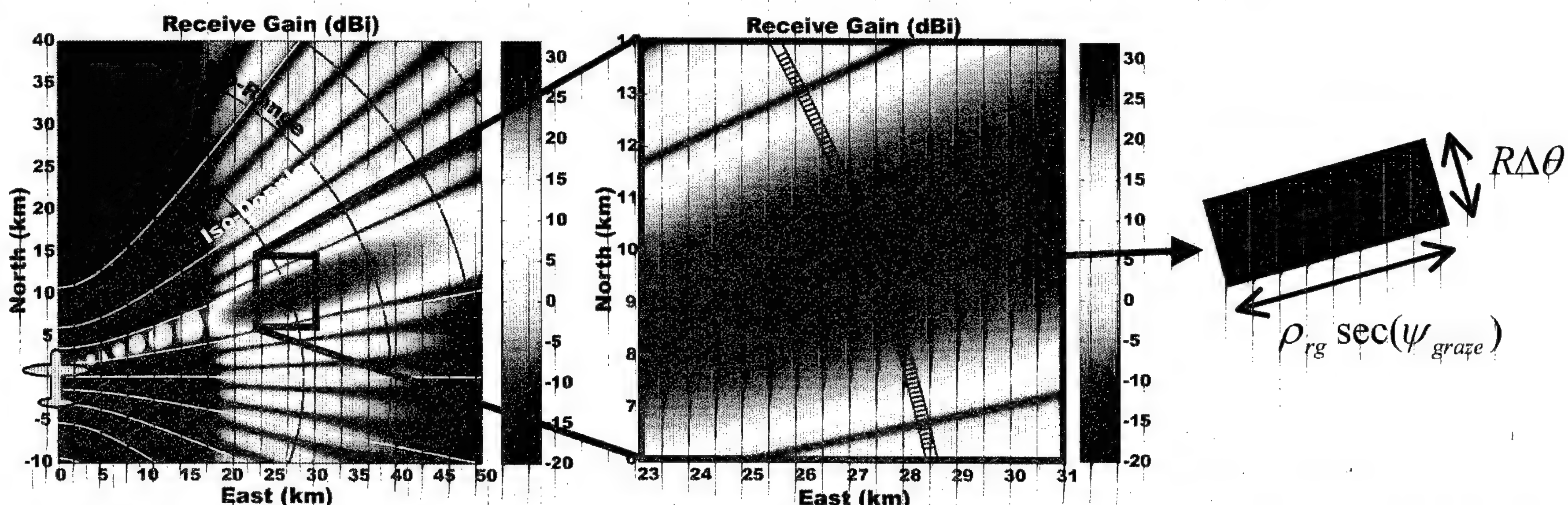
$$\mathbf{H}_{EOP} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \Lambda \\ \mathbf{a}(\vartheta) & \mathbf{0} & \Lambda \\ \mathbf{0} & \mathbf{0} & \Lambda \\ \mathbf{0} & \mathbf{a}(\vartheta) & \Lambda \\ \mathrm{M} & \mathrm{M} & \mathrm{O} \end{bmatrix} \quad MN \times \frac{M}{2}$$

Clutter Model

- The clutter contribution to a range bin sample is a *coherent sum* of the returns from a large number of *independent* clutter patches.
- We treat the return from each clutter patch as a complex, zero mean random variable whose variance (energy) is given by the radar range equation:

$$E_{patch}(\theta) = P \times T_p \times G^{Tx}(\theta) \times \frac{1}{4\pi R^2} \times \bar{\sigma}_0 \times R\Delta\theta\rho_{rg}\sin(\psi_{graze}) \times \frac{1}{4\pi R^2} \times \frac{\lambda^2 G^{Rx}(\theta)}{4\pi}$$

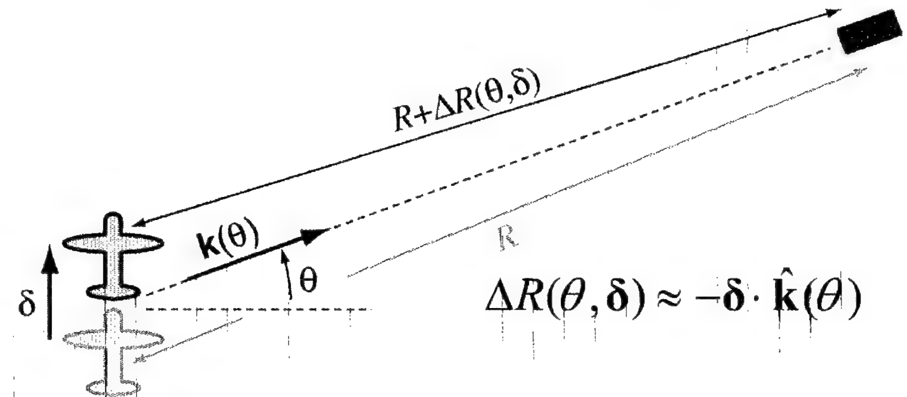
Patch Energy; Transmit Power; Pulse Width; TX Antenna Gain; Propagation Loss to Patch; Clutter Reflectivity; Patch Area; Propagation Loss to Radar; Antenna Effective Area



Clutter Correlation

The cross correlation between two pulses for a *single clutter patch* is

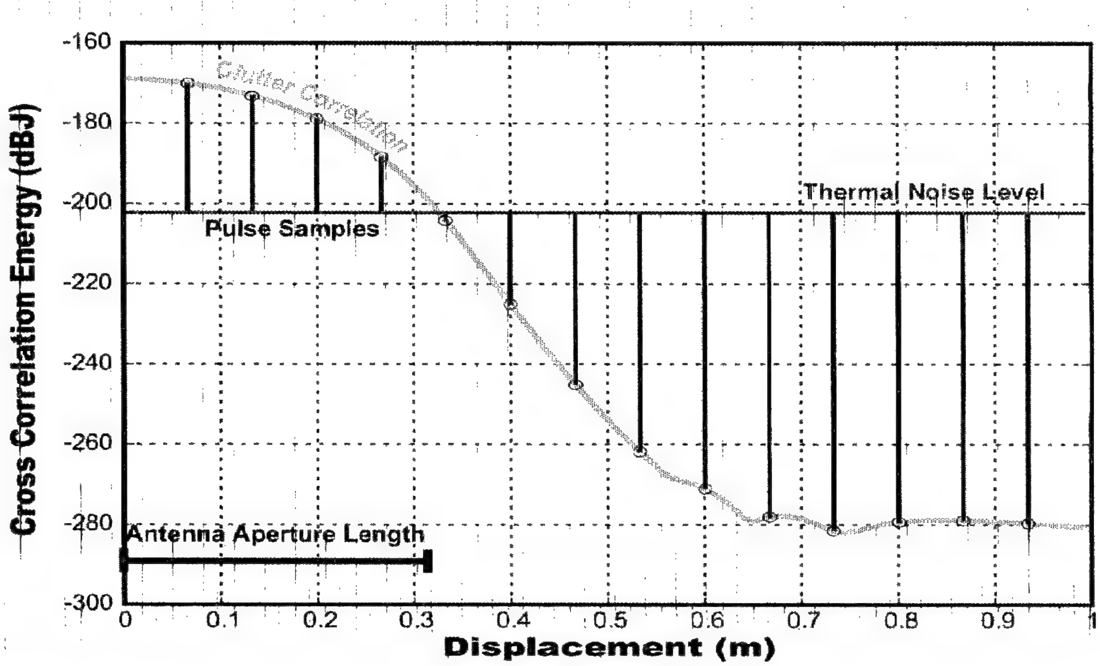
$$R_{patch}(\theta,\boldsymbol{\delta}) = E_{patch}(\theta)\exp\left(-2\pi i\frac{2\Delta R(\theta,\boldsymbol{\delta})}{\lambda}\right)$$



$$\Delta R(\theta,\boldsymbol{\delta}) \approx -\boldsymbol{\delta}\cdot\hat{\mathbf{k}}(\theta)$$

***Total* clutter correlation between pulses as a function of displacement is**

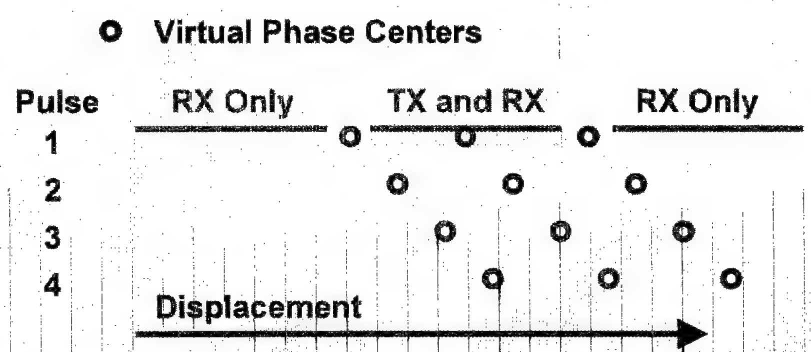
$$R_{clutter}(\boldsymbol{\delta}) = \sum_{\theta} R_{patch}(\theta,\boldsymbol{\delta})$$



RF Tags MIMO Channel

Interference Covariance Matrix

Interference Covariance Matrix



Clutter covariance depends only on the relative displacements of the receiver virtual phase centers.

$$\delta_{\Delta m,\Delta n} = \Delta m V_{AC}\, PRI + \Delta n \frac{d}{2}$$

$$[\mathbf{R_Z}]_{(m_1,n_1),(m_2,n_2)} = R_{clutter}(\delta_{m_2-m_1,n_2-n_1}) + \delta(m_2-m_1)\delta(n_2-n_1)E_{thermal}$$

Thermal Energy Per Sample

(Boltzmann's equation)

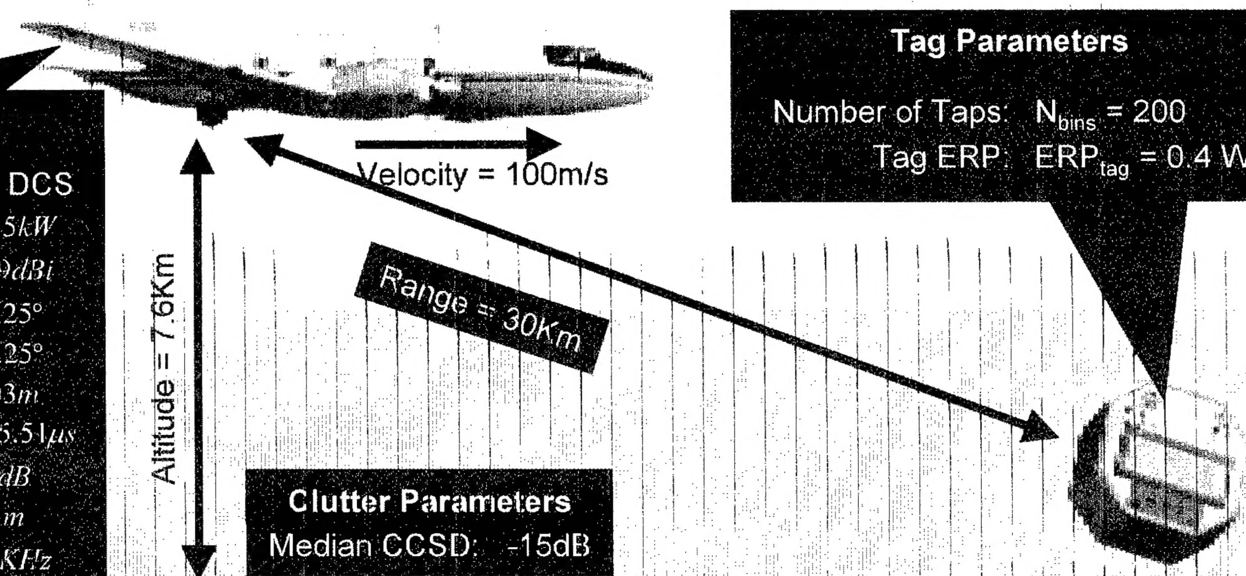
$$\underbrace{E_{thermal}}_{\text{Thermal Energy}} = \underbrace{k}_{\text{Boltzmann's Constant}} \times \underbrace{T}_{\text{Temperature}} \times \underbrace{F_n}_{\text{Receiver Noise Figure}}$$

Simulation Parameters

Fixed Value
Nominal Value

Radar Parameters

Platform:	Veridian DCS
Transmit Power:	$P_t^{radar} = 3.5kW$
Transmit/Receive Gain:	$G^{radar} = 29dBi$
Azimuth Beamwidth:	$\beta_{az} = 5.25°$
Elevation Beamwidth:	$\beta_{el} = 6.25°$
Wavelength:	$\lambda = .03m$
Pulse Width:	$T_r = 45.51\mu s$
Noise Figure:	$F_n = 2dB$
Range Resolution:	$\rho_{rg} = .3m$
PRF:	$PRF = 2KHz$

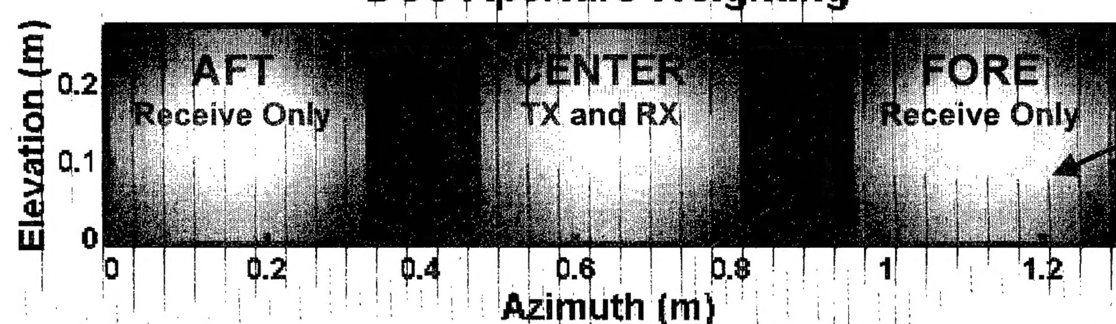


Tag Parameters

Number of Taps: $N_{bins} = 200$
Tag ERP: $ERP_{tag} = 0.4$ W

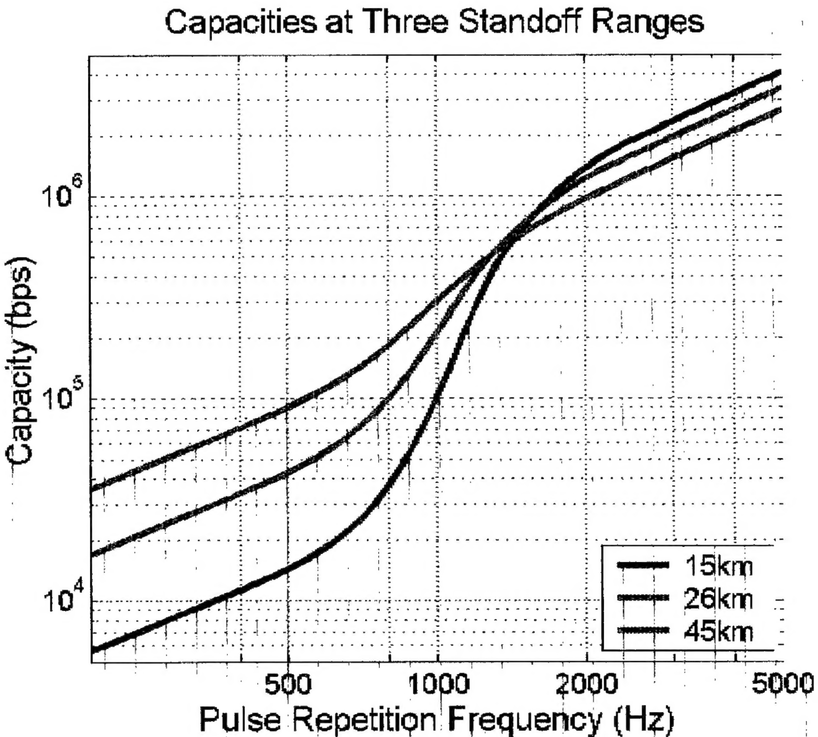
Clutter Parameters
Median CCSD: -15dB

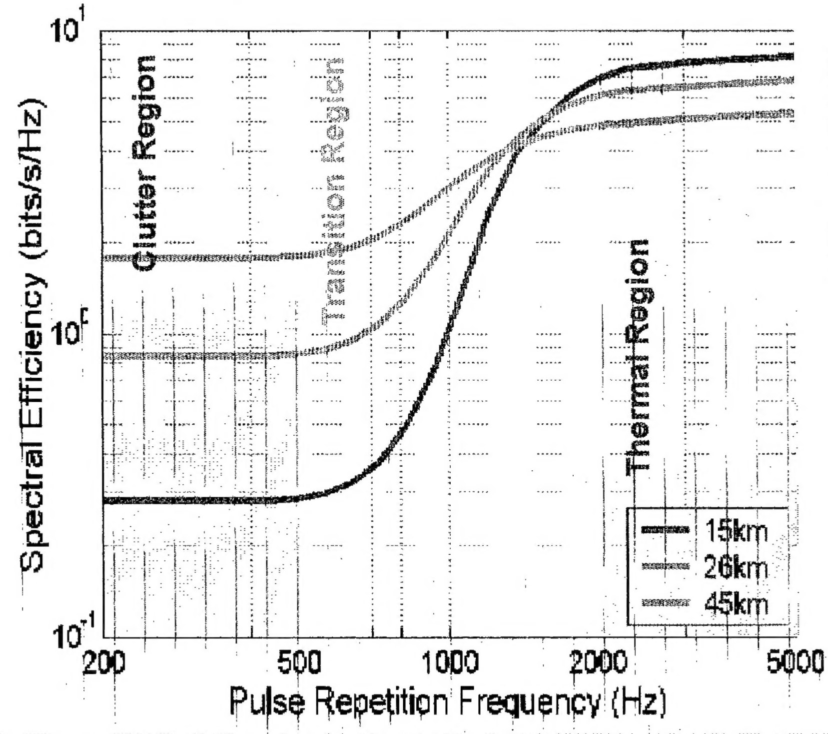
DCS Aperture Weighting



Single Receiver Capacities

Three Standoff Ranges





Model
UT-EOP

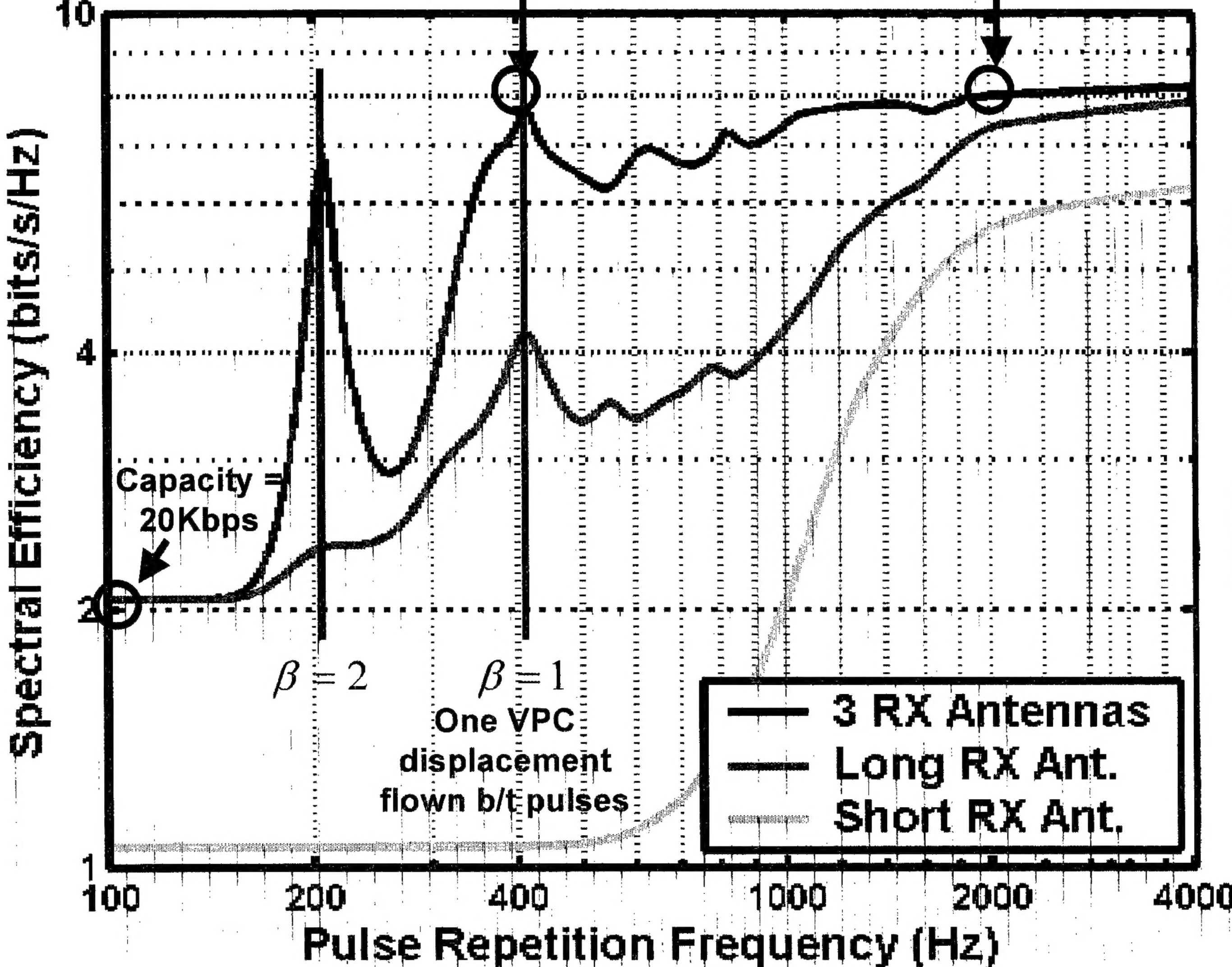
Res.
0.3 m

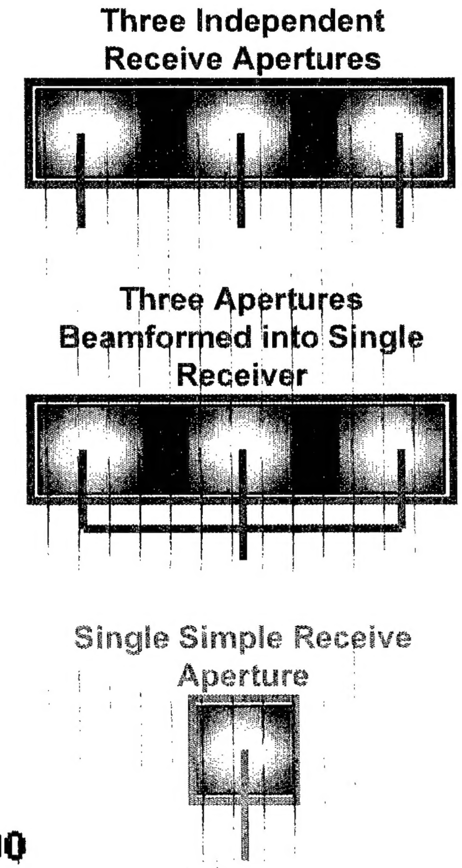
Single Receiver Observations

- **Spectral efficiency is strongly influenced by PRF.**
- **The dominant source of residual interference (clutter vs. thermal) is determined by the radar PRF.**
- **In clutter dominated PRF region, capacity goes *up* with longer ranges.**
- **In thermal dominated PRF region, capacity goes *down* with longer ranges.**

DCS / RF Tag Spectral Efficiency

Uninformed Transpond Every Other Pulse





Summary

- RF tags combines elements of MIMO communications and GMTI STAP.
- Channel capacities of >1Mbps are possible for representative radar and tag systems.
- A simple, general formula for the capacity of an injective MIMO channel was derived and used in calculating the channel capacity of a multiple receiver radar system.
- Multiple receive channel radars outperform single receive channel radars in medium PRF operation.
- High spectral efficiencies are possible even at low PRFs for a multiple receiver radar provided certain "DPCA-like" conditions are met.